9.3.1 WORK AND ENERGY[M34]

9.3.1.1 Work

When we push against a brick wall, nothing happens. We have applied a force, but the wall has not yielded and shows no effects. On the other hand, when we apply exactly the same force to a stone, the stone flies through the air for some distance. In the latter case something ahs been accomplished because of our push, while in the former there has been no result. What is the essential difference between these two situations? In the first case, where we pushed against a wall, our hand did not move; the force remained in place. But in the second case, where we threw the stone, our hand *did* move while the force was being applied and before the stone left our hand. The *motion* of the force was what made the difference.

If we think carefully along these lines, we will see that, whenever a force acts so as to produce motion in a body, the force itself undergoes a displacement. In order to make this notion definite, a physical quantity called work is defined as follows:

$$\mathbf{W} = \mathbf{F}s$$

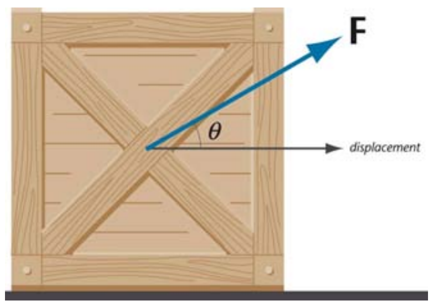
Whenever a force acts so as to produce motion in a body, the force itself undergoes a displacement. A physical quantity called work, **W**, is defined accordingly:

$$\mathbf{W} = \mathbf{F}s$$

where **F** is the force involved and s is the distance through which the force acts.

This definition tells us that, unless a force acts through a distance, no work is done, no matter how great the force. And even if a body moves through a distance, no work is done unless a force is acting upon it or it exerts a force on something else in the direction of movement.

When the force on an object and the object's displacement are in different directions, only the component of the force that is in the direction of the object's displacement does work. Accordingly, if the angle between the force and the direction of displacement is θ, as illustrated, the work done is given by:



$$\mathbf{W} = \mathbf{F}s(\cos\theta)$$

One common example of doing work is the lifting of an object against the force of gravity. It is easy to calculate the work done in this case because the force of gravity on a body of mass m is the same as its weight, m**g**. Thus, if the body is raised to a height h, we have:

$$\mathbf{F} = m\mathbf{g} \quad \text{and} \quad s = h$$

and so the work done is:

$$\mathbf{W} = \mathbf{F}s = m\mathbf{g}h$$

The SI unit of work is the same as that for energy, the **joule** (1 J = 1 N·m).

9.3.1.2 Potential Energy

We have already seen that potential energy is stored energy, energy that gives a body the capacity to do work. We will consider two specific examples of this type of energy: gravitational potential energy and elastic potential energy.

9.3.1.2.1 Gravitational Potential Energy

The gravitational potential energy of a mass of m kilograms lifted through a vertical height of h metres is precisely the work done in lifting the mass:

$$\textbf{Gravitational Potential Energy} = PE_g = mgh$$

If a mass of m kilograms is lifted from a height of h_1 metres to a height of h_2 metres, the work done, **W**, is simply the difference in energy between the two positions:

$$\mathbf{W} = mgh_2 - mgh_1$$

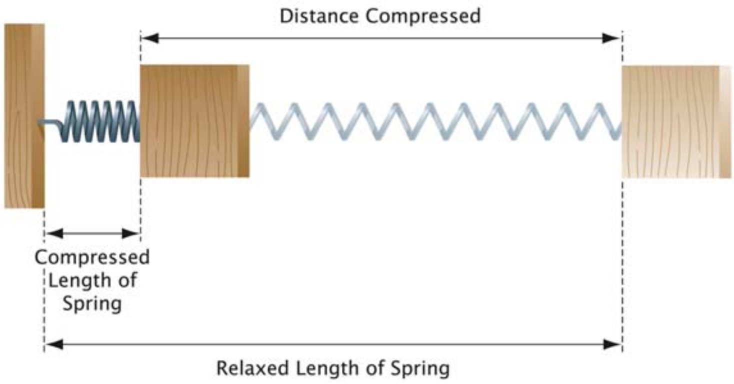
A more general statement of this relationship is:

Work Done = **Change in Energy**

and this applies to any change in energy.

9.3.1.2.2 Elastic Potential Energy

Elastic potential energy is energy that can be stored in a deformable object, such as the strings of a tennis racquet or guitar, or a simple spring. When a spring is stretched or compressed, elastic potential energy is stored in the spring, and it is capable of doing work as it returns to its *relaxed length*.



The Elastic Potential Energy in a spring is given by the equation:

$$\textbf{Elastic Potential Energy} = PE_{elastic} = \frac{1}{2}kx^2$$

where k is the spring constant and x is the distance the spring is stretched or compressed from its relaxed length. The spring constant for a flexible spring is small, while that of a stiff spring is large.

9.3.1.3 Kinetic Energy

The kinetic energy of a mass of m kilograms moving with a velocity of **v** metres per second metres is given by the equation:

$$\textbf{Kinetic Energy} = KE = \frac{1}{2}mv^2$$

Thomas Young (1773–1829) derived a similar formula in 1807, although he neglected to add the 1/2 to the front and he didn't use the words mass and weight with the same precision that we do today. Young called the quantity *energy*. Lord Kelvin (1824–1907) later added the adjective *kinetic* to differentiate it from *potential* energy, which had subsequently been described by William Rankine (1820–1872) in 1853[1].

[1] http://hypertextbook.com/physics/mechanics/energy-kinetic/

When a force is applied to an object, the work done by the force is equal to the change in energy of the object. Thus, for an object of mass m kg, accelerated on a horizontal surface from an initial velocity of **u** m/s to a final velocity of **v** m/s, the work done, **W**, is given by:

$$\mathbf{W} = \tfrac{1}{2}m\mathbf{v}^2 - \tfrac{1}{2}m\mathbf{u}^2$$

We can derive this equation for Work from our original Work equation and one of our equations of motion (see Section 7.2.2.6):

$$\mathbf{v}^2 = \mathbf{u}^2 + 2\mathbf{a}s \;\Rightarrow\; \mathbf{a}s = \frac{\mathbf{v}^2 - \mathbf{u}^2}{2}$$

$$\therefore\; \mathbf{W} = \mathbf{F}s = m\mathbf{a}s = \frac{m(\mathbf{v}^2 - \mathbf{u}^2)}{2} = \tfrac{1}{2}m\mathbf{v}^2 - \tfrac{1}{2}m\mathbf{u}^2$$

9.3.1.4 Conservation of Energy

The Law of Conservation of Energy guarantees that energy in a closed system will be conserved. In general, this condition is defined by the equation:

$$ME_{initial} = ME_{final}$$

In a mechanical system, however, not all energy is *useful*. In the context of machines, gravitational and elastic forces are considered conservative forces—they provide forms of energy that are available for conversion into mechanical energy. In the same context, forces such as friction are non-conservative forces—they produce a form of energy (heat) that cannot be converted into mechanical energy. As noted above, dissipative forces such as friction thus result in a loss of useable energy to the system.

Another way to distinguish between conservative and dissipative forces is to observe the relationship between the force and the path over which it acts. In the case of a conservative force, the work done and energy involved are completely independent of the length of the path, provided the paths have the same end point.

For example, in the illustration of gravitational potential energy in Figure 9.3.1.2 the gravitational potential energy of both men is the same, provided they have equal masses, even though one of them travelled a greater distance than the other one. The mass of the men and their height above the reference level are the only factors necessary for the determination of gravitational potential energy.

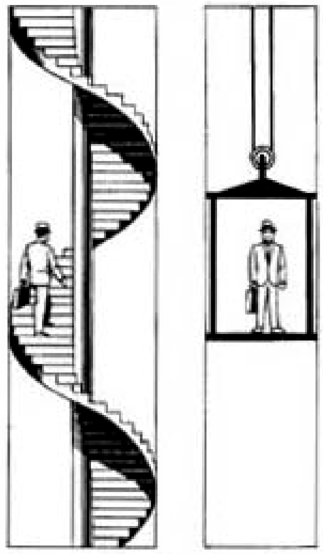


Figure 9.3.1.2 The potential energy of the man is dependent only on his mass and his height above the reference level

The amount of heat energy lost through friction can be quite different for two objects moving through the same height, however. Where the path is longer, the amount of mechanical energy that is converted to heat energy will be greater. This is true even if the frictional forces were to remain constant. Thus the work done by a given dissipative force varies directly with the length of the object path.

Nonetheless, the mechanical energy in a system is the sum of kinetic energy and all forms of potential energy in that system:

$$ME = KE + \Sigma PE$$

Thus, the Law of Conservation of Energy can be written as:

$$KE_{initial} + \Sigma PE_{initial} = KE_{final} + \Sigma PE_{final}$$

In the case where the only force acting is the force of gravity, we can expand this to:

$$\frac{1}{2}m\mathbf{v}_i^2 + m\mathbf{g}h_i = \frac{1}{2}m\mathbf{v}_f^2 + m\mathbf{g}h_f$$

9.3.1.6 Power

Power is the measure of how quickly something does work or changes energy from one form to another and is defined by the equation:

$$\mathbf{P} = \frac{\mathbf{W}}{t}$$

where **W** is the work done; and
t is the period of time over which the work is done.

Substituting the equation for work, we can also derive the equation for power in terms of force as:

$$\mathbf{P} = \frac{\mathbf{W}}{t} = \frac{\mathbf{F}s}{t} = \mathbf{F}\frac{s}{t}$$

i.e.

$$\mathbf{P} = \mathbf{F}\mathbf{v}$$

The SI derived unit of power is the **watt** ($1\ W = 1\ J{\cdot}s^{-1}$)

References

Holt Physics, Serway, R.A. and Faughn, J.S. (Holt, Rinehart and Winston, 2000) [ISBN 0-03-056544-8] Ch. 5

Work directly from text, with exercises:

5 Work and Energy

5.1 Work

5.2 Energy

5.3 Conservation of energy

5.4 Power